

Giant thermopower in superconducting heterostructures with spin-active interfaces

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Abstract

We predict parametrically strong enhancement of the thermoelectric effect in metallic bilayers consisting of two superconductors separated by a spin-active interface. The physical mechanism for such an enhancement is directly related to electron-hole imbalance generated by spin-sensitive quasiparticle scattering at the interface between superconducting layers. We explicitly evaluate the thermoelectric currents flowing in the system and demonstrate that they can reach maximum values comparable to the critical ones for superconductors under consideration.

Keywords: Superconductivity, thermoelectric effect, spin-dependent electron scattering, electron-hole imbalance

1. Introduction

It is well known that application of a thermal gradient ∇T to a normal conductor along with electric field \mathbf{E} results in the electric current

$$\mathbf{j} = \sigma_N \mathbf{E} + \alpha_N \nabla T, \quad \alpha_N \sim (\sigma_N/e)(T/\varepsilon_F). \quad (1)$$

Here σ_N defines Drude conductivity, α_N is thermoelectric coefficient and ε_F is the Fermi energy. Provided a metal is brought into a superconducting state, Eq. (1) is no longer correct since the electric field cannot penetrate into the bulk of a superconductor. Instead, one finds

$$\mathbf{j} = \mathbf{j}_s + \alpha_S \nabla T, \quad (2)$$

where \mathbf{j}_s is a supercurrent and α_S defines thermoelectric coefficient in a superconducting state. It turns out that by applying thermal gradient to a uniform superconductor it is not possible to induce and measure any current since thermal current would always be compensated by the supercurrent $\mathbf{j}_s = -\alpha_S \nabla T$. The way out is to consider non-uniform superconducting structures in which case no such compensation generally occurs [1, 2] and the thermoelectric current can be detected experimentally. Making use of this idea the thermoelectric effect was indeed demonstrated in several experiments with bimetallic superconducting rings [3, 4, 5]. Quite surprisingly, the magnitude of the effect was found to be several orders of magnitude bigger than predicted by theory [6]. The authors of a very recent experimental work [7] also observed a discrepancy between theory and their experimental data.

By now it is well understood that a small theoretical value of the thermoelectric coefficient in ordinary superconductors [6] $\alpha_S \sim \alpha_N$ is directly linked to the assumption that electron-hole symmetry remains preserved

in these structures. In this case contributions to the thermoelectric current provided by electron-like and hole-like excitations are of the opposite sign and almost cancel each other. Then, like in a normal metal, one inevitably finds that α_S is controlled by a parametrically small factor $T/\varepsilon_F \ll 1$.

The situation may change if for some reason the electron-hole symmetry gets violated. In this case – as it was demonstrated by a number of authors – a much stronger thermoelectric effect can be expected. The proposed mechanisms for the electron-hole symmetry violation and the related thermoelectric effect enhancement are diverse. In conventional superconductors doped by magnetic impurities, the presence of Andreev bound states formed near such impurities may yield an asymmetry between electron and hole scattering rates which in turn results in a drastic enhancement of the thermoelectric effect [8]. Likewise, the formation of quasi-bound Andreev states near non-magnetic impurities in unconventional superconductors may lead to much larger values of α_S in such systems [9]. Substantial enhancement of thermoelectric currents was also predicted in three terminal hybrid ferromagnet-superconductor-ferromagnet (FSF) [10] as well as in FS junctions in the presence of a Zeeman spin-splitting field [11].

In a recent work [12] we argued that the thermoelectric effect can be strongly enhanced also in metallic bilayers consisting of a superconductor and a normal metal (SN) provided these two metals are separated by a thin spin-active interface. By exactly solving the corresponding Bogolyubov-de-Gennes equations we evaluated the wave functions for electron-like and hole-like excitations in such systems demonstrating that spin-sensitive scattering at the SN interface can generate electron-hole imbalance and result in the presence of large thermoelectric currents in such

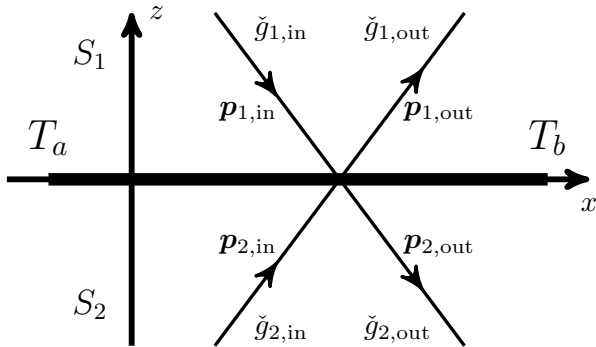


Figure 1: The system under consideration consisting of two superconducting layers S_1 and S_2 separated by a spin-active interface. The left and right edges of this superconducting bilayer are maintained at temperatures T_a and T_b , respectively. We also schematically indicate the quasiclassical electron Green functions for incoming and outgoing momentum values. These Green functions are matched at the spin-active interface by means of the proper boundary conditions as specified in the text.

systems. In this paper we will further extend our arguments [12] to superconducting multilayers with spin-active interfaces and demonstrate that thermoelectric properties of such systems may drastically differ from the those of bulk superconductors. As a simple example of such systems below we will specifically consider a superconductor with a thin ferromagnetic interlayer. We will show that provided a temperature gradient is applied along this interlayer the system develops a thermoelectric current which maximum values can be as high as the critical (depairing) current of a superconductor.

The structure of our paper is as follows. In Sec. 2 we will specify our model and outline our basic quasiclassical formalism of Eilenberger equations to be employed in our analysis of the thermoelectric effect. In Sec. 3 we will present an efficient method enabling one to derive the solution of these equations for the system under consideration. With the aid of this solution we will then derive a general expression for the thermoelectric current and also briefly discuss our results in Sec. 4.

2. The model and quasiclassical formalism

In what follows we will consider an extended metallic bilayer consisting of the two superconducting slabs S_1 and S_2 as shown in Fig. 1. We will assume that both metals are brought into direct contact with each other via a spin-active interface that is located in the plane $z = 0$. Such an interface can be formed, e.g., by an ultrathin layer of a ferromagnet. Our goal is to evaluate an electric current response to a temperature gradient applied to the system along the S_1S_2 interface. This temperature gradient is achieved by setting the temperature T at the left ($x \rightarrow -\infty$) and right ($x \rightarrow \infty$) edges of the bilayer equal respectively to T_a and T_b , see Fig. 1. For the sake of simplicity below we will assume that the temperature depends only on x and does not vary along y - and z -directions.

Within the quasiclassical theory of superconductivity [13], the current density $\mathbf{j}(\mathbf{r})$ in our system can be evaluated by means of the standard formula

$$\mathbf{j}(\mathbf{r}) = -\frac{eN_0}{8} \int d\varepsilon \langle \mathbf{v}_F \text{Sp}[\hat{\tau}_3 \hat{g}^K(\mathbf{p}_F, \mathbf{r}, \varepsilon)] \rangle, \quad (3)$$

where N_0 is the density of state at the Fermi level, $\mathbf{p}_F = m\mathbf{v}_F$ is the electron Fermi momentum vector, $\hat{\tau}_3$ is the Pauli matrix in the Nambu space, the angular brackets $\langle \dots \rangle$ denote averaging over the Fermi momentum directions and \hat{g}^K is the Keldysh block of the quasiclassical Green-Eilenberger function matrix

$$\check{g} = \begin{pmatrix} \hat{g}^R & \hat{g}^K \\ 0 & \hat{g}^A \end{pmatrix}. \quad (4)$$

Here and below the “hat”-symbol denotes 4×4 matrices in the Nambu \otimes Spin space while the “check”-symbol labels 8×8 matrices in the Keldysh \otimes Nambu \otimes Spin space.

The matrix function \check{g} obeys the transport-like Eilenberger equation [13]

$$[\varepsilon \hat{\tau}_3 - \check{\Delta}(\mathbf{r}), \check{g}] + i\mathbf{v}_F \nabla \check{g}(\mathbf{p}_F, \mathbf{r}, \varepsilon) = 0 \quad (5)$$

as well as the normalization condition

$$\check{g}^2 = 1. \quad (6)$$

The order parameter matrix $\check{\Delta}$ has only “retarded” and “advanced” components,

$$\check{\Delta} = \begin{pmatrix} \hat{\Delta} & 0 \\ 0 & \hat{\Delta} \end{pmatrix}, \quad \hat{\Delta} = \begin{pmatrix} 0 & \Delta \sigma_0 \\ -\Delta^* \sigma_0 & 0 \end{pmatrix}, \quad (7)$$

where σ_0 is the unity matrix in the spin space and Δ is the superconducting order parameter. As soon as we are interested in the electronic transport along the interface we set the phase difference between the two superconductors S_1 and S_2 to zero. Under this assumption order parameter can be made to be real everywhere in the system.

As usually, the quasiclassical equations (5) should be supplemented by boundary conditions which describe electron transfer across the SFS -interface by matching the Green function matrices \check{g} for incoming and outgoing momentum directions at both sides of this interface, see Fig. 1. In the case of spin-active interfaces the corresponding boundary conditions were derived in [14]. Here we will employ an equivalent approach [15].

The simplest model of the spin-active interface is described by three parameters, i.e. the transmission probabilities for opposite spin directions D_\uparrow and D_\downarrow as well as the so-called spin mixing angle θ . Previously we have already made use of this model, e.g., while considering crossed Andreev reflection in three-terminal FSF structures [16] or triplet pairing and dc Josephson effect in SFS junctions [17]. For simplicity we assume that the above three parameters do not depend on the sign of the quasiparticle momentum along the interface, i.e. $D_\uparrow(\mathbf{p}_\parallel) =$

$D_{\uparrow}(-\mathbf{p}_{\parallel})$ and so on. Then the elements of the interface scattering matrices for electrons

$$\mathcal{S} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \quad (8)$$

and holes

$$\underline{\mathcal{S}} = \begin{pmatrix} \underline{S}_{11} & \underline{S}_{12} \\ \underline{S}_{21} & \underline{S}_{22} \end{pmatrix} \quad (9)$$

take the form

$$S_{11} = S_{22} = \sqrt{R_{\sigma}} e^{i\theta_{\sigma}/2}, \quad (10)$$

$$S_{12} = S_{21} = i\sqrt{D_{\sigma}} e^{i\theta_{\sigma}/2}, \quad (11)$$

$$\underline{S}_{11} = \underline{S}_{22} = \sqrt{R_{-\sigma}} e^{-i\theta_{\sigma}/2}, \quad (12)$$

$$\underline{S}_{12} = \underline{S}_{21} = i\sqrt{D_{-\sigma}} e^{-i\theta_{\sigma}/2}, \quad (13)$$

where θ_{σ} is 2×2 diagonal matrix in the spin space defined as $\theta_{\sigma} = \theta\sigma_3$. The matrices $D_{\pm\sigma}$ and $R_{\pm\sigma}$ are composed of transmission and reflection probabilities for opposite spin directions as

$$D_{\sigma} = \begin{pmatrix} D_{\uparrow} & 0 \\ 0 & D_{\downarrow} \end{pmatrix}, \quad D_{-\sigma} = \begin{pmatrix} D_{\downarrow} & 0 \\ 0 & D_{\uparrow} \end{pmatrix}, \quad (14)$$

$$R_{\sigma} = \begin{pmatrix} R_{\uparrow} & 0 \\ 0 & R_{\downarrow} \end{pmatrix}, \quad R_{-\sigma} = \begin{pmatrix} R_{\downarrow} & 0 \\ 0 & R_{\uparrow} \end{pmatrix}, \quad (15)$$

where we defined the spin-up and spin-down reflection coefficients respectively as $R_{\uparrow} = 1 - D_{\uparrow}$ and $R_{\downarrow} = 1 - D_{\downarrow}$.

3. Riccati parameterization

In order to proceed we will employ the so-called Riccati parameterization of the retarded and advanced Green functions [18, 19].

$$\hat{g}^{R,A} = \pm \hat{N}^{R,A} \begin{pmatrix} 1 + \gamma^{R,A} \tilde{\gamma}^{R,A} & 2\gamma^{R,A} \\ -2\tilde{\gamma}^{R,A} & -1 - \tilde{\gamma}^{R,A} \gamma^{R,A} \end{pmatrix}, \quad (16)$$

where $\hat{N}^{R,A}$ represent the following matrices

$$\hat{N}^{R,A} = \begin{pmatrix} (1 - \gamma^{R,A} \tilde{\gamma}^{R,A})^{-1} & 0 \\ 0 & (1 - \tilde{\gamma}^{R,A} \gamma^{R,A})^{-1} \end{pmatrix}. \quad (17)$$

Here the Riccati amplitudes $\gamma^{R,A}$, $\tilde{\gamma}^{R,A}$ are 2×2 matrices in the spin space.

Parameterization of the Keldysh Green function involves two distribution functions [19] x^K , \tilde{x}^K also being 2×2 matrices in the spin space, namely

$$\hat{g}^K = 2\hat{N}^R \begin{pmatrix} x^K - \gamma^R \tilde{x}^K \tilde{\gamma}^A & -\gamma^R \tilde{x}^K + x^K \gamma^A \\ -\tilde{\gamma}^R x^K + \tilde{x}^K \tilde{\gamma}^A & \tilde{x}^K - \tilde{\gamma}^R x^K \gamma^A \end{pmatrix} \hat{N}^A. \quad (18)$$

The amplitudes $\gamma^{R,A}$, $\tilde{\gamma}^{R,A}$ obey the Riccati equations

$$i\mathbf{v}_F \nabla \gamma^{R,A} = (1 \quad \gamma^{R,A}) \hat{h} \begin{pmatrix} -\gamma^{R,A} \\ 1 \end{pmatrix}, \quad (19)$$

$$i\mathbf{v}_F \nabla \tilde{\gamma}^{R,A} = (\tilde{\gamma}^{R,A} \quad 1) \hat{h} \begin{pmatrix} 1 \\ -\tilde{\gamma}^{R,A} \end{pmatrix}, \quad (20)$$

while the distribution functions x^K and \tilde{x}^K satisfy the transport-like equations

$$i\mathbf{v}_F \nabla x^K = x^K (1 \quad 0) \hat{h} \begin{pmatrix} 1 \\ -\gamma^A \end{pmatrix} - (1 \quad \gamma^R) \hat{h} \begin{pmatrix} 0 \\ 1 \end{pmatrix} x^K, \quad (21)$$

and

$$i\mathbf{v}_F \nabla \tilde{x}^K = \tilde{x}^K (0 \quad 1) \hat{h} \begin{pmatrix} -\gamma^A \\ 1 \end{pmatrix} - (\tilde{\gamma}^R \quad 1) \hat{h} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tilde{x}^K, \quad (22)$$

where $\hat{h} = \varepsilon \hat{\tau}_3 - \hat{\Delta}(\mathbf{r})$.

Below it will be convenient for us to employ the parameterization of the Green function matrices in terms of the Riccati amplitudes γ , $\tilde{\gamma}$, Γ , $\tilde{\Gamma}$ as well as the distribution functions x , \tilde{x} , X , \tilde{X} (all being 2×2 matrices in the spin space) [19],

$$\check{g}_{i,\text{in}} = \check{g}_{i,\text{in}}[\gamma_i^R, \tilde{\Gamma}_i^R, \Gamma_i^A, \tilde{\gamma}_i^A, x_i, \tilde{X}_i], \quad i = 1, 2, \quad (23)$$

$$\check{g}_{i,\text{out}} = \check{g}_{i,\text{out}}[\Gamma_i^R, \tilde{\gamma}_i^R, \gamma_i^A, \tilde{\Gamma}_i^A, X_i, \tilde{x}_i], \quad i = 1, 2. \quad (24)$$

Boundary conditions [15] allow to express the interface values of the capital functions Γ and X in terms of the lower-case functions γ and x . For Riccati amplitudes $\Gamma_1^{R,A}$ and $\tilde{\Gamma}_1^{R,A}$ at the interface we obtain

$$\Gamma_1^R(0) = \tilde{\Gamma}_1^R(0) = \mathcal{M}_1^R e^{i\theta_{\sigma}} \left\{ \gamma_1^R(0) \sqrt{R_{\uparrow} R_{\downarrow}} + \gamma_2^R(0) \sqrt{D_{\uparrow} D_{\downarrow}} - \gamma_1^R(0) [\gamma_2^R(0)]^2 e^{i\theta_{\sigma}} \right\}, \quad (25)$$

$$\Gamma_1^A(0) = \tilde{\Gamma}_1^A(0) = \mathcal{M}_1^A e^{-i\theta_{\sigma}} \left\{ \gamma_1^A(0) \sqrt{R_{\uparrow} R_{\downarrow}} + \gamma_2^A(0) \sqrt{D_{\uparrow} D_{\downarrow}} - \gamma_1^A(0) [\gamma_2^A(0)]^2 e^{-i\theta_{\sigma}} \right\}, \quad (26)$$

where

$$\mathcal{M}_1^R = \left\{ 1 - [\gamma_2^R(0)]^2 \sqrt{R_{\uparrow} R_{\downarrow}} e^{i\theta_{\sigma}} - \gamma_2^R(0) \gamma_1^R(0) \sqrt{D_{\uparrow} D_{\downarrow}} e^{i\theta_{\sigma}} \right\}^{-1}, \quad (27)$$

and

$$\mathcal{M}_1^A = \left\{ 1 - [\gamma_2^A(0)]^2 \sqrt{R_{\uparrow} R_{\downarrow}} e^{-i\theta_{\sigma}} - \gamma_2^A(0) \gamma_1^A(0) \sqrt{D_{\uparrow} D_{\downarrow}} e^{-i\theta_{\sigma}} \right\}^{-1}. \quad (28)$$

The interface values of the Riccati amplitudes $\Gamma_2^{R,A}$ and $\tilde{\Gamma}_2^{R,A}$ can be obtained from Eqs. (25)-(28) by interchanging the indices $1 \leftrightarrow 2$. Here we used the fact that for the real order parameter one has $\gamma_i^{R,A} = \tilde{\gamma}_i^{R,A}$. From Eqs. (25)-(28) we observe that within the adopted model for the spin-active interface this equality also holds for the capital Riccati amplitudes $\Gamma_i^{R,A} = \tilde{\Gamma}_i^{R,A}$.

In the same way we can express the interface values of the distribution functions. At the S_1 -side of the interface we have

$$\begin{aligned}
X_1(0) = & \mathcal{M}_1^R \mathcal{M}_1^A \left(x_1^K(0) \left\{ \sqrt{R_\sigma} - [\gamma_2^R(0)]^2 \sqrt{R_{-\sigma}} e^{i\theta_\sigma} \right\} \right. \\
& \times \left\{ \sqrt{R_\sigma} - [\gamma_2^A(0)]^2 \sqrt{R_{-\sigma}} e^{-i\theta_\sigma} \right\} \\
& + x_2^K(0) \left\{ \sqrt{D_\sigma} - \gamma_1^R(0) \gamma_2^R(0) \sqrt{D_{-\sigma}} e^{i\theta_\sigma} \right\} \\
& \times \left\{ \sqrt{D_\sigma} - \gamma_1^A(0) \gamma_2^A(0) \sqrt{D_{-\sigma}} e^{-i\theta_\sigma} \right\} \\
& - \tilde{x}_2^K(0) \left\{ \gamma_1^R(0) \sqrt{R_\sigma D_{-\sigma}} - \gamma_2^R(0) \sqrt{R_{-\sigma} D_\sigma} \right\} \\
& \times \left. \left\{ \gamma_1^A(0) \sqrt{R_\sigma D_{-\sigma}} - \gamma_2^A(0) \sqrt{R_{-\sigma} D_\sigma} \right\} \right), \quad (29)
\end{aligned}$$

and

$$\begin{aligned}
\tilde{X}_1(0) = & \mathcal{M}_1^R \mathcal{M}_1^A \left(\tilde{x}_1^K(0) \left\{ \sqrt{R_{-\sigma}} - [\gamma_2^R(0)]^2 \sqrt{R_\sigma} e^{i\theta_\sigma} \right\} \right. \\
& \times \left\{ \sqrt{R_{-\sigma}} - [\gamma_2^A(0)]^2 \sqrt{R_\sigma} e^{-i\theta_\sigma} \right\} \\
& + \tilde{x}_2^K(0) \left\{ \sqrt{D_{-\sigma}} - \gamma_1^R(0) \gamma_2^R(0) \sqrt{D_\sigma} e^{i\theta_\sigma} \right\} \\
& \times \left\{ \sqrt{D_{-\sigma}} - \gamma_1^A(0) \gamma_2^A(0) \sqrt{D_\sigma} e^{-i\theta_\sigma} \right\} \\
& - x_2^K(0) \left\{ \gamma_1^R(0) \sqrt{R_{-\sigma} D_\sigma} - \gamma_2^R(0) \sqrt{R_\sigma D_{-\sigma}} \right\} \\
& \times \left. \left\{ \gamma_1^A(0) \sqrt{R_{-\sigma} D_\sigma} - \gamma_2^A(0) \sqrt{R_\sigma D_{-\sigma}} \right\} \right). \quad (30)
\end{aligned}$$

The interface values for the distribution functions $X_2(0)$ and $\tilde{X}_2(0)$ can be recovered from Eqs. (29) and (30) simply by interchanging the indices $1 \leftrightarrow 2$. Within our simple model all Riccati amplitudes and distribution functions are diagonal in the spin space. The coordinate dependence of the distribution function can be easily found. We obtain

$$x_i = [1 - \gamma_i^R \gamma_i^A] \times \begin{cases} h_a, & v_x > 0, \\ h_b, & v_x < 0, \end{cases} \quad (31)$$

$$\tilde{x}_i = -[1 - \gamma_i^R \gamma_i^A] \times \begin{cases} h_b, & v_x > 0, \\ h_a, & v_x < 0, \end{cases}, \quad (32)$$

$$X_i = X_i(0) \frac{1 - \Gamma_i^R \Gamma_i^A}{1 - \Gamma_i^R(0) \Gamma_i^A(0)}, \quad (33)$$

$$\tilde{X}_i = \tilde{X}_i(0) \frac{1 - \Gamma_i^R \Gamma_i^A}{1 - \Gamma_i^R(0) \Gamma_i^A(0)}, \quad (34)$$

where the functions $h_{a,b}$ are related to the equilibrium (Fermi) distribution function with temperatures $T_{a,b}$, i.e.

$$h_{a,b} = \tanh \frac{\varepsilon}{2T_{a,b}}. \quad (35)$$

4. Thermoelectric current

Let us apply the above quasiclassical formalism in order to derive the thermoelectric current flowing along the spin-active interface in the ballistic limit. According to Eq. (3) the component of the current density along the interface is expressed in terms of the combination $\text{Sp}(\hat{\tau}_3 \hat{g}_{\text{in}}^K + \hat{\tau}_3 \hat{g}_{\text{out}}^K)$. The above combination is evaluated by solving the Eilenberger equations (5), (6) supplemented by the proper boundary conditions at the $S_1 S_2$ interface. This task can be conveniently accomplished employing the Riccati parameterization of the Green functions [18, 19]. Making use of the results of the previous section we obtain

$$\begin{aligned}
\text{Sp}(\hat{\tau}_3 \hat{g}_{i,\text{in}}^K + \hat{\tau}_3 \hat{g}_{i,\text{out}}^K) = & 2 \text{Sp} \left[\frac{(x_i - \tilde{x}_i)(1 + \Gamma_i^R \Gamma_i^A)}{(1 - \gamma_i^R \Gamma_i^R)(1 - \gamma_i^A \Gamma_i^A)} \right] \\
& + 2 \text{Sp} \left[\frac{(X_i - \tilde{X}_i)(1 + \gamma_i^R \gamma_i^A)}{(1 - \gamma_i^R \Gamma_i^R)(1 - \gamma_i^A \Gamma_i^A)} \right], \quad i = 1, 2. \quad (36)
\end{aligned}$$

With the aid of Eqs. (31)-(33) one can rewrite the above expression in the form

$$\begin{aligned}
\text{Sp}(\hat{\tau}_3 \hat{g}_{i,\text{in}}^K + \hat{\tau}_3 \hat{g}_{i,\text{out}}^K) = & 2 \text{Sp} \left[\frac{(1 - \gamma_i^R \gamma_i^A)(1 + \Gamma_i^R \Gamma_i^A)}{(1 - \gamma_i^R \Gamma_i^R)(1 - \gamma_i^A \Gamma_i^A)} (h_a + h_b) \right] \\
& + 2 \text{Sp} \left[\frac{(1 - \Gamma_i^R \Gamma_i^A)(1 + \gamma_i^R \gamma_i^A)}{(1 - \gamma_i^R \Gamma_i^R)(1 - \gamma_i^A \Gamma_i^A)} \frac{X_i(0) - \tilde{X}_i(0)}{1 - \Gamma_i^R(0) \Gamma_i^A(0)} \right]. \quad (37)
\end{aligned}$$

Eq. (37) can further be simplified if one observes that the four functions γ_i^R , $1/\gamma_i^A$, $1/\Gamma_i^R$ and Γ_i^A obey the same Riccati equations implying that the combination

$$\frac{1 - \Gamma_i^R \Gamma_i^A}{1 - \Gamma_i^R \gamma_i^R} \frac{1 - \gamma_i^A \gamma_i^R}{1 - \gamma_i^A \Gamma_i^A}$$

is spatially constant, i.e. it does not depend on the coordinates. Then one can rewrite Eq. (37) as

$$\begin{aligned}
\text{Sp}(\hat{\tau}_3 \hat{g}_{i,\text{in}}^K + \hat{\tau}_3 \hat{g}_{i,\text{out}}^K) = & 2 \text{Sp} \left[\frac{(1 - \gamma_i^R \gamma_i^A)(1 + \Gamma_i^R \Gamma_i^A)}{(1 - \gamma_i^R \Gamma_i^R)(1 - \gamma_i^A \Gamma_i^A)} \right] \\
& \times (h_a + h_b) + 2 \frac{1 + \gamma_i^R \gamma_i^A}{1 - \gamma_i^R \gamma_i^A} \\
& \times \text{Sp} \left\{ \frac{[1 - \gamma_i^A(0) \gamma_i^R(0)] [X_i(0) - \tilde{X}_i(0)]}{[1 - \gamma_i^R(0) \Gamma_i^R(0)] [1 - \gamma_i^A(0) \Gamma_i^A(0)]} \right\}. \quad (38)
\end{aligned}$$

What remains is to find the difference of the distribution functions $X_i(0) - \tilde{X}_i(0)$ on both sides of the spin-active interface. Making use of Eqs. (29)-(30), we obtain

$$\begin{aligned}
X_i(0) - \tilde{X}_i(0) = & \mathcal{M}_i^R \mathcal{M}_i^A \\
& \times [A_i(h_a - h_b) \sigma_3 \text{sgn } v_x + B_i(h_a + h_b)], \quad i = 1, 2, \quad (39)
\end{aligned}$$

where

$$A_1 = (R_\downarrow - R_\uparrow) K_2 [\gamma_1^R(0)\gamma_1^A(0) - \gamma_2^R(0)\gamma_2^A(0)], \quad (40)$$

$$\begin{aligned} B_1 = & K_2 [1 + \gamma_1^R(0)\gamma_1^A(0)\gamma_2^R(0)\gamma_2^A(0)] \\ & + (R_\uparrow + R_\downarrow) K_1 \gamma_2^R(0)\gamma_2^A(0) \\ & - \sqrt{R_\uparrow R_\downarrow} K_1 \left\{ [\gamma_2^R(0)]^2 e^{i\theta_\sigma} + [\gamma_2^A(0)]^2 e^{-i\theta_\sigma} \right\} \\ & - \sqrt{D_\uparrow D_\downarrow} K_2 \left\{ \gamma_1^R(0)\gamma_2^R(0)e^{i\theta_\sigma} + \gamma_1^A(0)\gamma_2^A(0)e^{-i\theta_\sigma} \right\} \\ & - R_\uparrow R_\downarrow K_2 [\gamma_1^R(0)\gamma_1^A(0) + \gamma_2^R(0)\gamma_2^A(0)] \\ & - \sqrt{R_\uparrow R_\downarrow D_\uparrow D_\downarrow} K_2 [\gamma_1^R(0)\gamma_2^A(0) + \gamma_2^R(0)\gamma_1^A(0)]. \quad (41) \end{aligned}$$

Here we defined

$$K_i = 1 - \gamma_i^R(0)\gamma_i^A(0), \quad i = 1, 2. \quad (42)$$

Analogous expressions can also be derived for A_2 and B_2 . Combining Eqs. (38)-(41) with the general expression (3) and observing that the terms containing the combination $h_a + h_b$ do not contribute to the current and defining the unity vector in the x -direction \mathbf{e}_x , we arrive at the final result for the current in the S_1 superconductor

$$\begin{aligned} \mathbf{j}(z > 0) = & \mathbf{e}_x \frac{eN_0}{2} \int d\varepsilon \left[\tanh \frac{\varepsilon}{2T_a} - \tanh \frac{\varepsilon}{2T_b} \right] \\ & \times \left\langle \theta(v_x)\theta(v_z)v_x(R_\uparrow - R_\downarrow) \right. \\ & \left. [1 - \gamma_2^R(0)\gamma_2^A(0)] [1 - \gamma_1^R(0)\gamma_1^A(0)] \frac{1 + \gamma_1^R(z)\gamma_1^A(z)}{1 - \gamma_1^R(z)\gamma_1^A(z)} \right. \\ & \left. \times [\gamma_1^R(0)\gamma_1^A(0) - \gamma_2^R(0)\gamma_2^A(0)] \text{Sp}(\sigma_3 \mathcal{P}) \right\rangle, \quad (43) \end{aligned}$$

where

$$\begin{aligned} \mathcal{P} = & \left| 1 - [\gamma_1^R(0)]^2 \sqrt{R_\uparrow R_\downarrow} e^{i\theta_\sigma} - [\gamma_2^R(0)]^2 \sqrt{R_\uparrow R_\downarrow} e^{i\theta_\sigma} \right. \\ & \left. - 2\gamma_2^R(0)\gamma_1^R(0)\sqrt{D_\uparrow D_\downarrow} e^{i\theta_\sigma} + [\gamma_2^R(0)\gamma_1^R(0)]^2 e^{2i\theta_\sigma} \right|^{-2}. \quad (44) \end{aligned}$$

The current density in the second superconductor $\mathbf{j}(z < 0)$ is trivially derived from Eq. (43) by interchanging the indices $1 \leftrightarrow 2$.

Note that at subgap energies we have $\gamma_i^R(z)\gamma_i^A(z) \equiv 1$ and, hence, the fraction $[1 - \gamma_i^R(0)\gamma_i^A(0)]/[1 - \gamma_i^R(z)\gamma_i^A(z)]$ in Eq. (43) becomes indefinite. In this case with the aid of Eqs. (19) one can establish an equivalent representation for the above fraction

$$\begin{aligned} & \frac{1 - \gamma_i^R(0)\gamma_i^A(0)}{1 - \gamma_i^R(z)\gamma_i^A(z)} \\ & = \exp \left(-\frac{i \text{sgn } z}{|v_z|} \int_0^z \Delta(z') [\gamma_i^R(z') - \gamma_i^A(z')] dz' \right), \quad (45) \end{aligned}$$

which remains regular at subgap energies.

The above equations defining the thermoelectric current density $\mathbf{j}(z)$ flowing in each of the two superconductors S_1 and S_2 along the spin-active interface represent the main result of this work. It is easy to verify that provided the superconducting order parameter in one of the superconductors ($\Delta(z > 0)$ or $\Delta(z < 0)$) tends to zero, Eq. (43) reduces to the result for the thermoelectric current in an SN bilayer derived previously [12] within the framework of a different technique.

Let us briefly analyze the above results. To begin with we observe that the thermoelectric current (43) may differ from zero only in asymmetric structures consisting of *different* superconductors S_1 and S_2 . Furthermore – similarly to [12] – the current (43) vanishes if at least one of the two conditions, $D_\uparrow = D_\downarrow$ or $\theta = 0$, is fulfilled. On the other hand, provided both these conditions are simultaneously violated, the thermoelectric current differs from zero and its value can become large.

It is also worth pointing out that the thermoelectric currents (43), (44) flow in the *opposite* directions in the superconductors S_1 and S_2 . In each of the superconductors the current density depends on the coordinate z in the vicinity of the interface and tends to some nonzero values far from it. The latter feature is specific for our ballistic model within which the elastic mean free path ℓ tends to infinity and no electron momentum relaxation occurs. Assuming the mean free path to be finite (which is always the case in any real metal), one can demonstrate that the thermoelectric current density $\mathbf{j}(z)$ remains appreciable only in the vicinity of the spin-active interface $|z| < \ell$ and decays exponentially into the superconducting bulk at distances from this interface exceeding the elastic mean free path. The corresponding analysis, however, goes beyond the frames of this work and will be made public elsewhere [20].

In order to accurately evaluate the general expression for the thermoelectric current defined in Eqs. (43), (44) one should selfconsistently determine the functions $\gamma_{1,2}^{R,A}(z)$ as well as the order parameter $\Delta(z)$ for any given values of the parameters D_\uparrow , D_\downarrow and θ . Technically this is a rather complicated task which can only be handled numerically. There is, however, no particular need in this calculation since the order-of-magnitude estimate of the current can easily be obtained directly from Eqs. (43), (44). The magnitude of the thermoelectric current density can roughly be estimated as

$$|\mathbf{j}(z)| \sim j_c (R_\uparrow - R_\downarrow) \sin \theta \frac{T_a - T_b}{T_c}, \quad (46)$$

where $j_c \sim ev_F N_0 T_c$ is the critical current of a clean superconductor with the critical temperature T_c . Hence, we observe that, in contrast to the standard situation [6], our result (46) does not contain the small factor $T/\varepsilon_F \ll 1$, i.e. the magnitude of the thermoelectric effect becomes really large in our case. For instance, by setting $(R_\uparrow - R_\downarrow) \sin \theta \sim 1$ and $T_1 - T_2 \sim T_c$, one achieves

the thermoelectric current densities of the same order as the critical one j_c .

To conclude, we demonstrated that quasiparticle scattering at spin-active interfaces generates electron-hole imbalance in superconductors which may yield an enhancement of thermoelectric currents in such structures up to values as high as the critical (depairing) current of a superconductor. The same effect is expected in more complicated (e.g. layered) superconducting structures containing spin-active interfaces. Such thermoelectric currents can easily be detected and investigated in modern experiments.

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